Image reconstruction algorithms for microtomography

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Microtomography systems

NDT systems

Small animal CT

Dental CBCT

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Nano x-ray microscopy

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Micro CT images

Detection of Osteolytic Lesions by Micro-CT is Superior to X-ray

Sterling et al

Stampanoni et al

Dental CBCT
Problems

- Object preparation, fixation, irradiation, etc
- Polychromatic source, miscalibrations, etc
- Small object size: insufficient absorption contrast
- Limited field-of-view, limited data, incomplete geometry
- Large amount of digital data
Solutions

• Region-of-interest reconstruction

• Fully 3D cone-beam scanning and reconstruction

• The use of phase contrast

• Software/hardware acceleration
Geometry

Synchrotron

Parallel beam
The source is far away from the object

Microfocus tube, microscopy

Cone beam
The source is close to the object:
- Increased flux
- Magnification
- Fully 3D
Inverse problem

Projection data: \( g_{\theta,s} = \int f \, dl \)

Radon transform

\[ Af = g \]

To find \( f \) from \( g \)?
Backprojection

Integration of the projection data over the whole range of $\theta$

$$A^* g = \frac{1}{\pi} \int_0^{\pi} g \ d\theta$$
Algorithms: classification

- Fourier algorithm
- Filtered backprojection (FBP)
- Backprojection and filtering (BPF)
- Iterative

Radon transform

\[ Af = g \]

\[ f = (A^* A)^{-1} A^* g \]

\[ = A^* (AA^*)^{-1} g \]

\[ = F_n^{-1} F_{n-1} g \]
Parallel-beam geometry (Synchrotron)
Fourier slice theorem
Image reconstruction with NFFT

\[ f = F_2^{-1} F_1 g \]

Interpolation from the polar grid to the Cartesian is required

Linogram ("pseudo-polar") grid

Nonequispaced Fast Fourier transform (NFFT) can be used

Potts et al, 2001
FBP and BPF algorithms

\[
f = \left( A^* A \right)^{-1} A^* g = A^* \left( A A^* \right)^{-1} g
\]

\[
F_2 \left[ \left( A^* A \right)^{-1} \delta \right] = \sqrt{\xi^2 + \eta^2}
\]

\[
F_1 \left[ \left( A A^* \right)^{-1} \delta \right] = |\xi|
\]

“ramp filter”

\[
f = \frac{1}{\pi} F_2^{-1} \left[ \sqrt{\xi^2 + \eta^2} \right] \otimes \otimes \int_0^\pi g_\theta d\theta
\]

\[
f = \frac{1}{\pi} \int_0^\pi F_1^{-1} [\xi] \otimes g_\theta d\theta
\]
FBP algorithm

\[ f = \int_{0}^{\pi} q \ast g_{\theta} d\theta \]
Local ("Lambda") tomography

\[ F_1^{-1} \left[ \xi | \hat{g} \right] = H \frac{\partial}{\partial s} g \]
\[ f = A^* H \frac{\partial}{\partial s} g \]

\[ \frac{\partial}{\partial s} g \] Local operator

\[ f_\Lambda = A^* \frac{\partial}{\partial s} g \]

Hilbert transform is non-local:

\[ H g(s) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(t)}{s-t} dt \]
Cone-beam geometry (Microfocus x-ray tube)
Feldkamp algorithm with a circular orbit

Feldkamp, Davis, Kress, 1984

\[ f = \int_0^\pi q \ast g_\theta d\theta \]
Kirillov-Tuy condition

Exact 3D reconstruction is possible if every plane through the object intersects the source trajectory at least once.

\[ g(a(\theta), u) = \int_{0}^{\infty} f(a(\theta) + su) \, ds \]

x-ray source trajectory; parametrized as \( a(\theta) \)
Circular source orbit: artifacts

Slices of 3D reconstruction of a phantom (cone angle 30 deg):

Bronnikov 1995, 2000
ROI reconstruction

Two-step data acquisition

Source

ROI

Sample

Position 1

Detector

Position 2

Sample
Non-planar source orbits

- two orthogonal circles
- two circles and line
- helix (most feasible mechanically)
- saddle

Non-planar orbits satisfy the Kirillov-Tuy condition, but special reconstruction algorithms are required.
Katsevich algorithm for a non-planar source orbit orbit

Katsevich, 2002

\[ f = \frac{1}{2\pi} A^* H_\pi \frac{\partial}{\partial \theta} g \]

1. Differentiation of data
2. Hilbert transform along the filtration lines inside the Tam-Danielsson window
3. Backprojection

PI-line ("segment", "chord") between \(a(\theta_1)\) and \(a(\theta_2)\): \(\theta_2 - \theta_1 = 2\pi\)

Helix: \( a(\theta) = \left( R \cos \theta, R \sin \theta, \frac{h\theta}{2\pi} \right)^T \)
BPF algorithms for ROI reconstruction

\[ f = \frac{1}{2\pi} H_\pi A_\pi^* \frac{\partial}{\partial \theta} g \]

1. Differentiation of data
2. Backprojection onto the PI chord (locality!)
3. Hilbert transform along the PI chord

Using that \( f \) has the finite support and

\[ HHg = -g \]

Zou, Pan, Sidky, 2005 derived:

\[ f = \frac{1}{2\pi} H_{\pi, a_1-a_2}^{-1} A_\pi^* \frac{\partial}{\partial \theta} g \]
Ultra-fast implementation

- Graphic card (GPU)
- CPU

Reconstruction of a 512x512x512 image from 360 projections:

<table>
<thead>
<tr>
<th>CPU (~2 GHz)</th>
<th>Single core</th>
<th>Dual core</th>
<th>Quad core</th>
<th>Twin quad-core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>~80 sec</td>
<td>~40 sec</td>
<td>~20 sec</td>
<td>~10 sec</td>
</tr>
</tbody>
</table>

Reconstruction of a 1024x1024x1024 image from 800 projections:

<table>
<thead>
<tr>
<th>CPU (~2 GHz)</th>
<th>Dual core</th>
<th>Twin quad-core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>~480 sec</td>
<td>~120 sec</td>
</tr>
</tbody>
</table>
Phase-contrast microtomography (Free propagation mode)
Phase contrast

Interference of the phase-shifted wave with the unrefracted waves
Inline phase-contrast imaging

Snigirev et al, 1995
Polychromatic x-ray phase contrast

Wilkins et al, 1996
Phase-contrast tomography with Radon inversion: edges
Inverse problem of phase-contrast microtomography

Object function: \( f = n - 1 \)

find \( f(x_1, x_2, x_3) \) from \( I^z_\theta(x, y), \ 0 \leq \theta < \pi \)

- CTF (Cloetens et al, 1999)
- TIE (Paganin and Nugent, 1998) \( \} \) Phase retrieval, more than one detection plane

- Weak-absorption TIE (Bronnikov, 1999) FBP, single detection plane
Radon transform solution of TIE

\[
I^d_\theta(x, y) = I^0_\theta \left[ 1 - \frac{\lambda d}{2\pi} \nabla^2 \varphi_\theta(x, y) \right]
\]

Bronnikov, 1999

\[
g_\theta(x, y) = \frac{I^d_\theta}{I_i} - 1
\]

\[
f = \frac{1}{4\pi^2 d} \iint \hat{g}_\theta(s, \omega) d\theta d\omega
\]
Phase-contrast reconstruction in the form of the FBP algorithm


\[ f = \frac{1}{4\pi^2 d} \int_0^\pi q \ast g_\theta d\theta \]

\[ Q = \frac{|\xi|}{\xi^2 + \eta^2} \]

\[ Q_\alpha = \frac{|\xi|}{\xi^2 + \eta^2 + \alpha} \]

Gureyev et al, 2004: choice of \( \alpha \) for linearly dependent absorption and refraction
Implementation at SLS

Phase tomography reconstruction (a) and the 3D rendering (b) of a 350 microns thin wood sample using modified filter given in the Eq. (8). The length of the scale bar is 50 µm.

Validation of the MBA method: (a) Phase tomographic reconstruction of sample consisting of polyacrylate, starch and cross-linked rubber matrix obtained using DPC and (b) using MBA. The length of the scale bar is 100 µm.

\[ Q_\alpha = \frac{|\xi|}{\xi^2 + \eta^2 + \alpha} \]

“MBA: Modified Bronnikov Algorithm”
Groso, Abela, Stampanoni, 2006
Implementation at Ghent University

De Witte, Boone, Vlassenbroeck, Dierick, and Van Hoorebeke, 2009

Radon inversion  “Modified Bronnikov Algorithm”  “Bronnikov-Aided Correction”
Polychromatic source, mixed phase and amplitude object

Air bubbles in epoxy, relatively strong absorption:

Reconstruction by the “Bronnikov Filter” with correction

Data provided by Xradia
Summary

• New developments in theory:
  - parallel-beam CT (synchrotron): the use of NFFT
  - cone-beam CT (microfocus): exact reconstruction with non-planar orbits; exact ROI reconstruction (Katsevich formula, PI line, Hilbert transform on chords)

• New developments in implementation:
  – ultra-fast 3D reconstruction on multicore processors
  – \((1024^3\) voxels within one-two minutes on a PC)

• New developments in coherent methods:
  - robust algorithms for 3D phase reconstruction
  - correction for the phase component