Image reconstruction algorithms for microtomography

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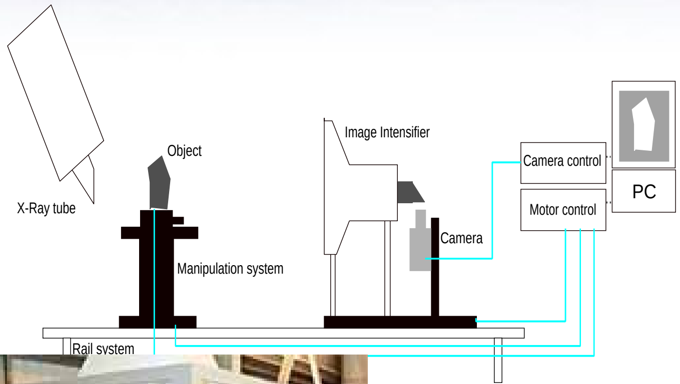
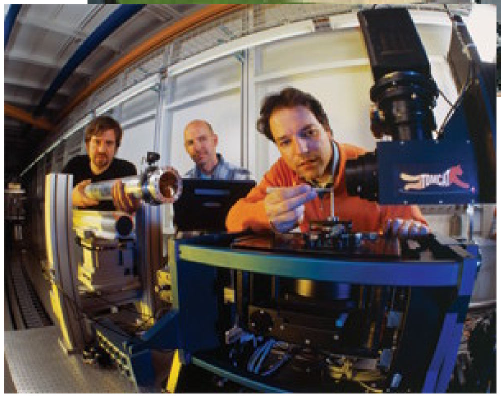
Microtomography systems



NDT systems

Desktop systems



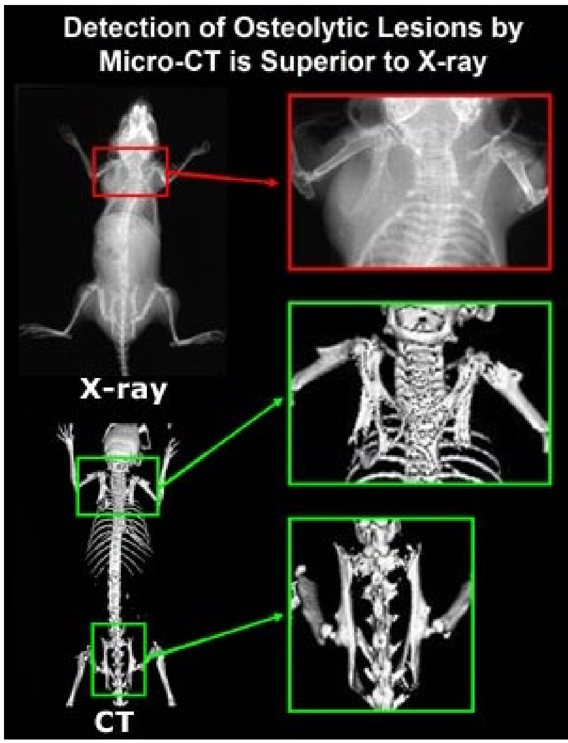
Synchrotron setup

Small animal CT

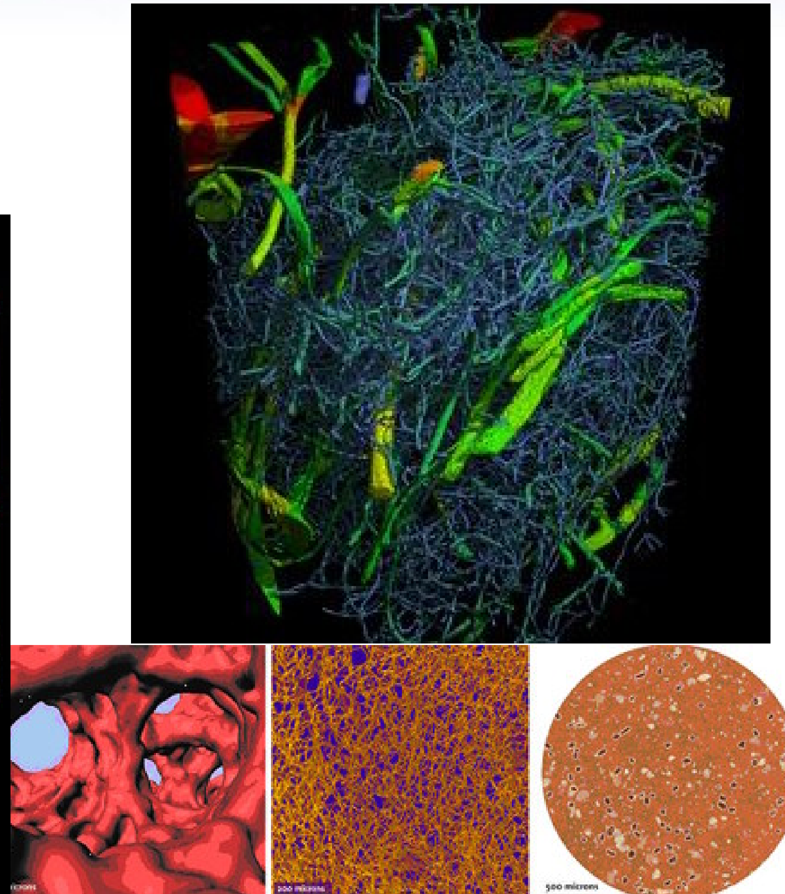
Dental CBCT

Nano x-ray microscopy

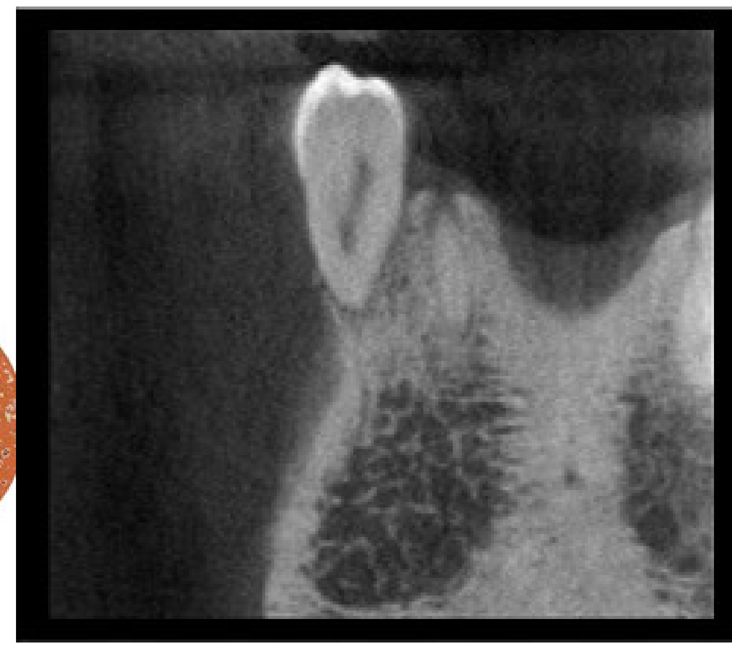
Micro CT images



Sterling *et al*



Stampanoni *et al*



Dental CBCT

Problems

- Object preparation, fixation, irradiation, etc
- Polychromatic source, miscalibrations, etc
- Small object size: insufficient absorption contrast
- Limited field-of-view, limited data, incomplete geometry
- Large amount of digital data

Solutions

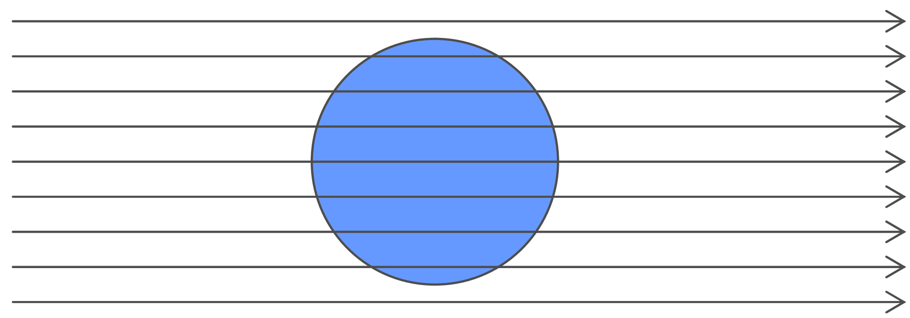
- Region-of-interest reconstruction
- Fully 3D cone-beam scanning and reconstruction
- The use of phase contrast
- Software/hardware acceleration

Geometry

Synchrotron

Parallel beam

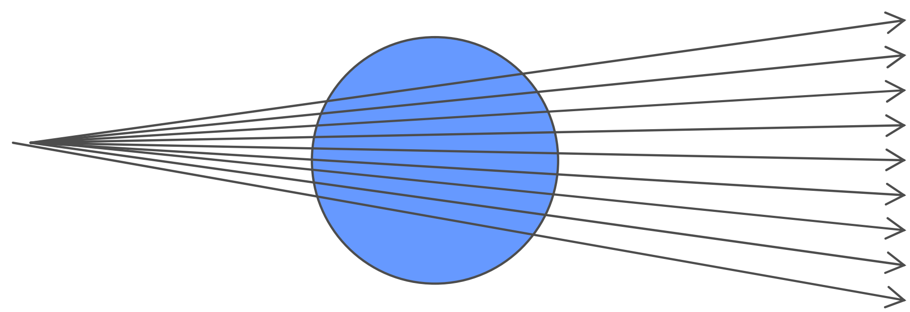
The source is far away from the object



Microfocus tube, microscopy

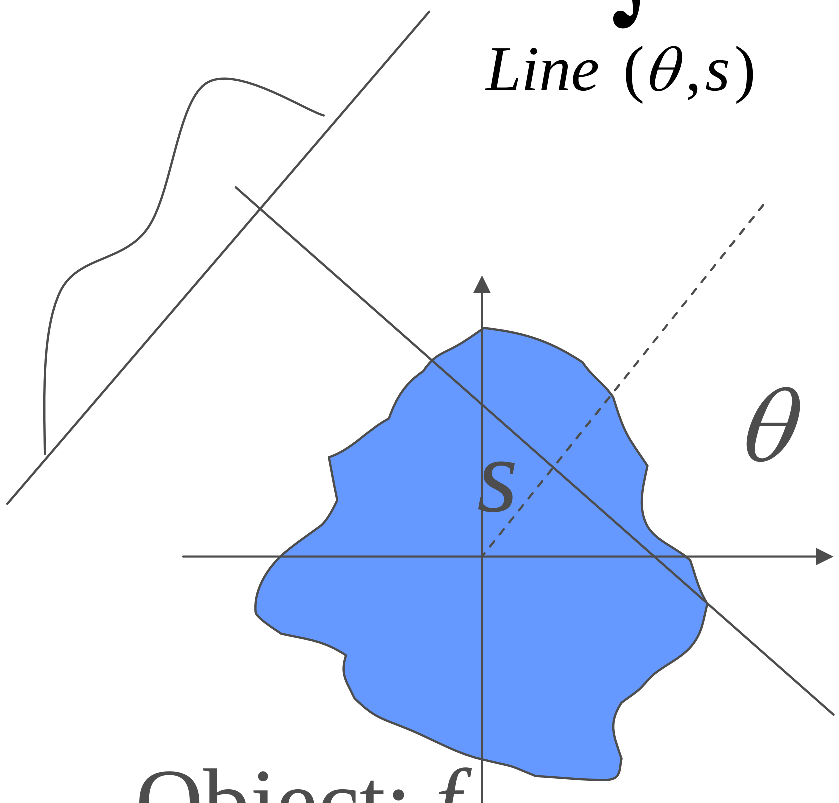
Cone beam

The source is close to the object:
- Increased flux
- Magnification
- Fully 3D



Inverse problem

Projection data: $g_{\theta,s} = \int\limits_{Line\ (\theta,s)} f \; dl$



Object: f

Radon transform

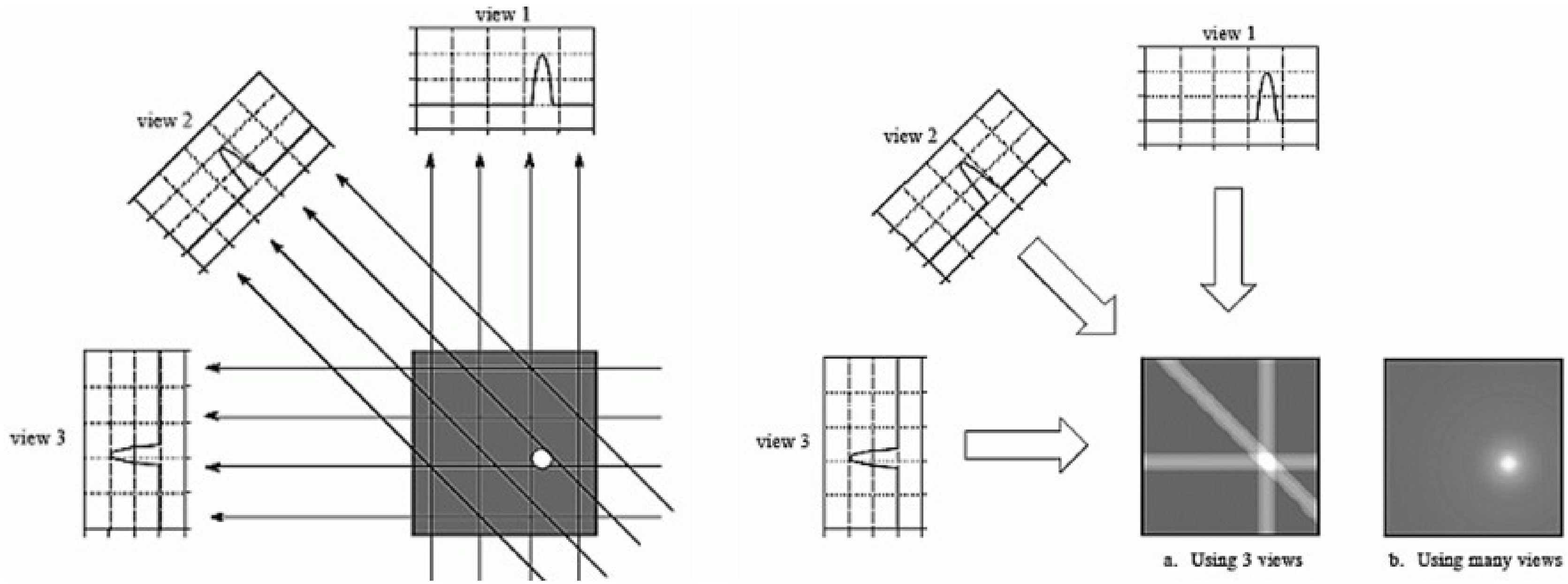
$$Af = g$$

To find f from g ?

Backprojection

Integration of the projection data over the whole range of θ

$$A^{*}g = \frac{1}{\pi}\int_{0}^{\pi} g \; d\theta$$



Algorithms: classification

- Fourier algorithm
- Filtered backprojection (FBP)
- Backprojection and filtering (BPF)
- Iterative

Radon transform

$$Af = g$$

Imaging equation

$$f = \left(A^* A\right)^{-1} A^* g$$

BPF

$$= A^* \left(A A^*\right)^{-1} g$$

FBP

$$= F_n^{-1} F_{n-1} g$$

Fourier

Parallel-beam geometry (Synchrotron)

Fourier slice theorem

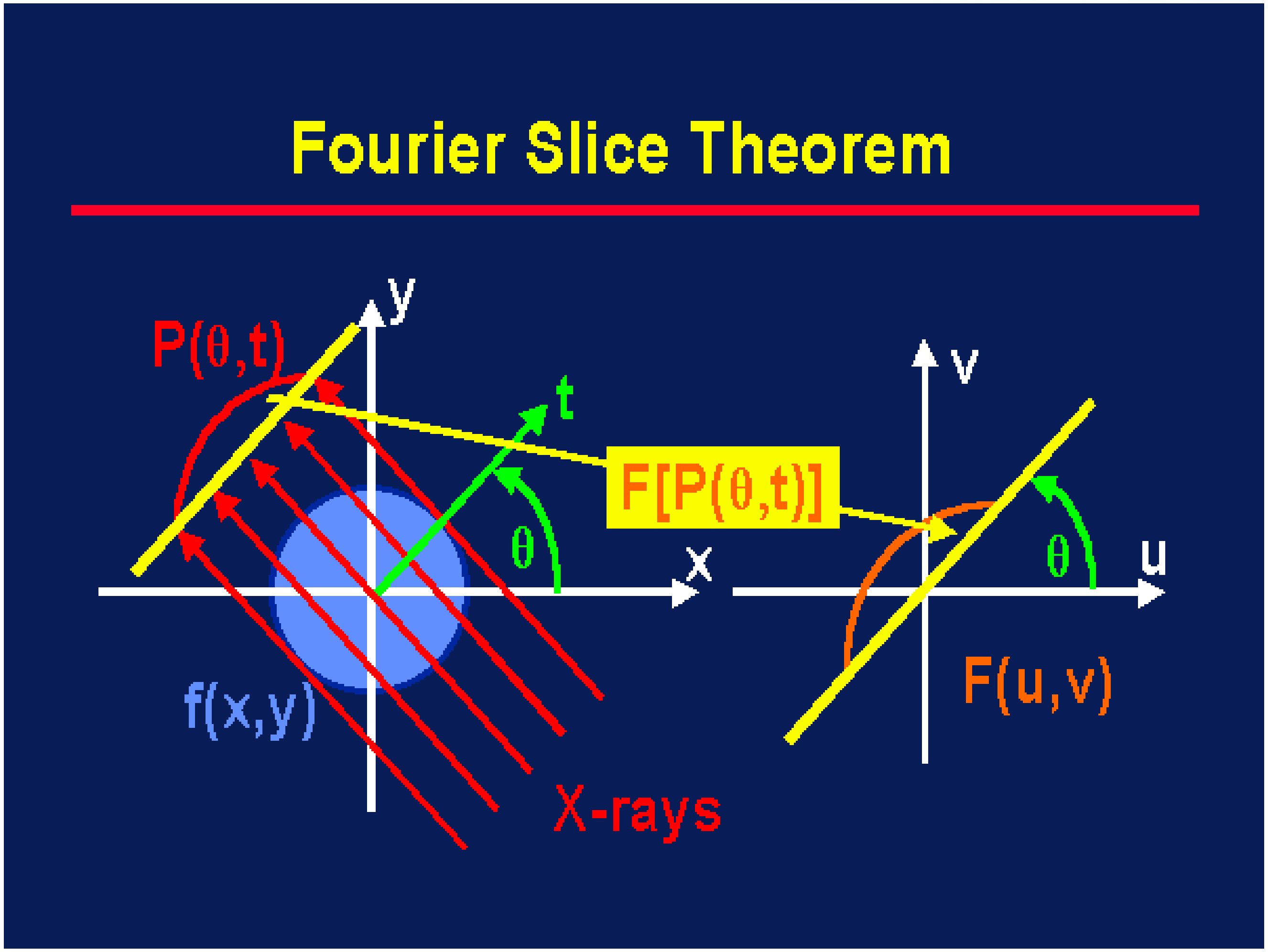
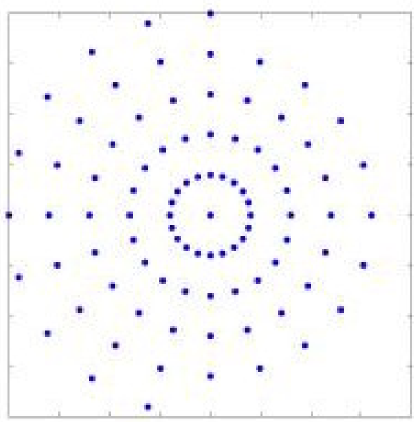


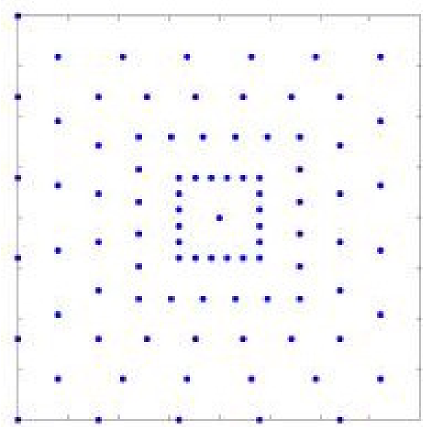
Image reconstruction with NFFT

$$f = F_2^{-1} F_1 g$$

Interpolation from the polar grid
to the Cartesian is required



Linogram ("pseudo-polar") grid



Nonequispaced Fast Fourier transform (NFFT) can be used

Potts *et al*, 2001

FBP and BPF algorithms

$$f = \left(A^{*}A\right)^{-1} A^{*} g \quad = A^{*}\left(AA^{*}\right)^{-1} g$$

$$F_2\left[\left(A^{*}A\right)^{-1}\delta\right] = \sqrt{\xi^2 + \eta^2}$$

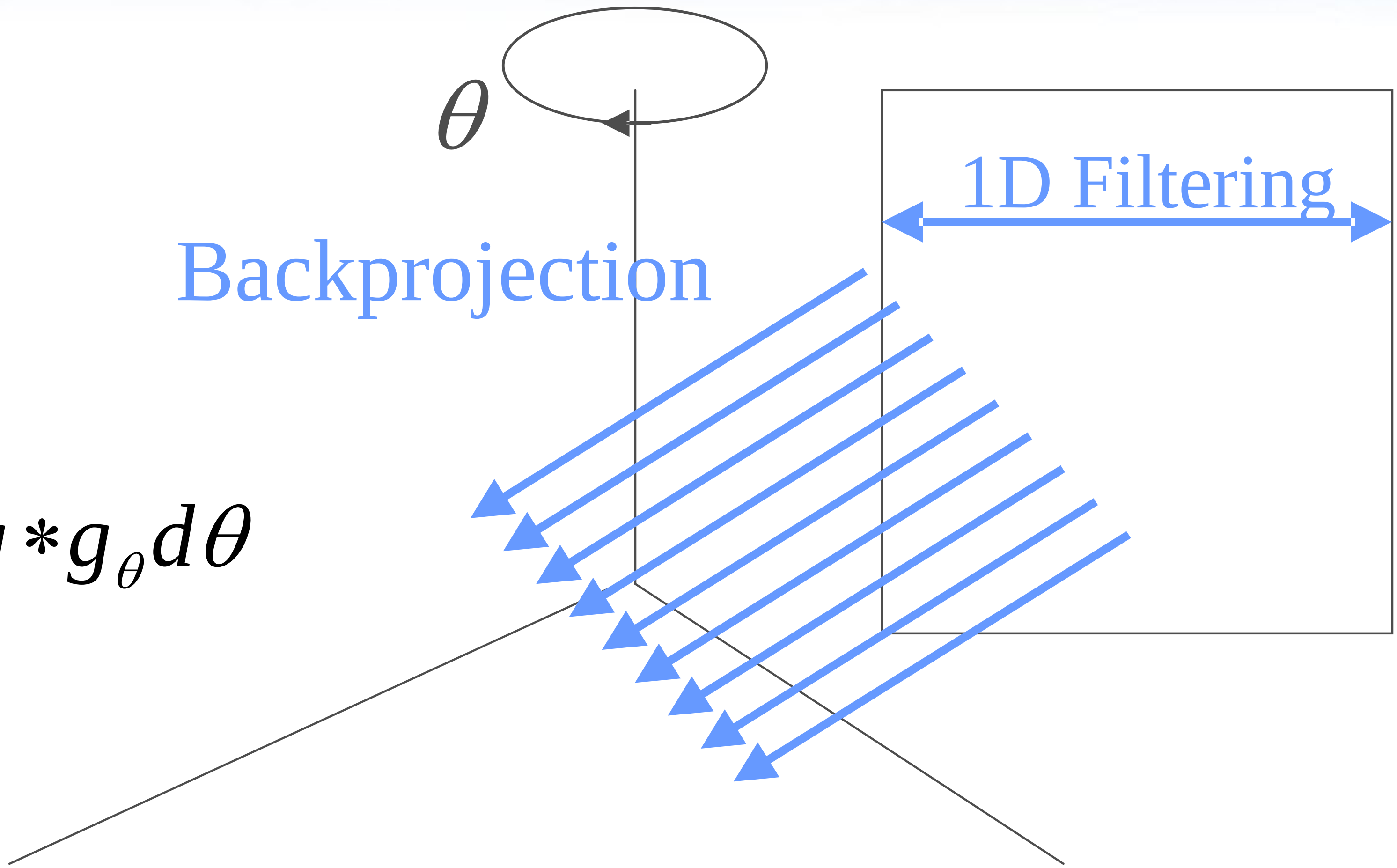
$$F_1\left[\left(AA^{*}\right)^{-1}\delta\right] = \left|\xi\right|$$

"ramp filter"

$$f = \frac{1}{\pi} F_2^{-1}\left[\sqrt{\xi^2 + \eta^2}\right] \otimes \otimes \int_0^{\pi} g_\theta d\theta$$

$$f = \frac{1}{\pi} \int_0^{\pi} F_1^{-1}\left[\left|\xi\right|\right] \otimes g_\theta d\theta$$

FBP algorithm



$$f = \int_{0}^{\pi} q * g_{\theta} d\theta$$

Local ("Lambda") tomography

$$F_1^{-1}\left[|\xi|\hat{g}\right] = H\frac{\partial}{\partial s}g \qquad f = A^* H\frac{\partial}{\partial s}g$$

$$\frac{\partial}{\partial s}g$$ Local operator

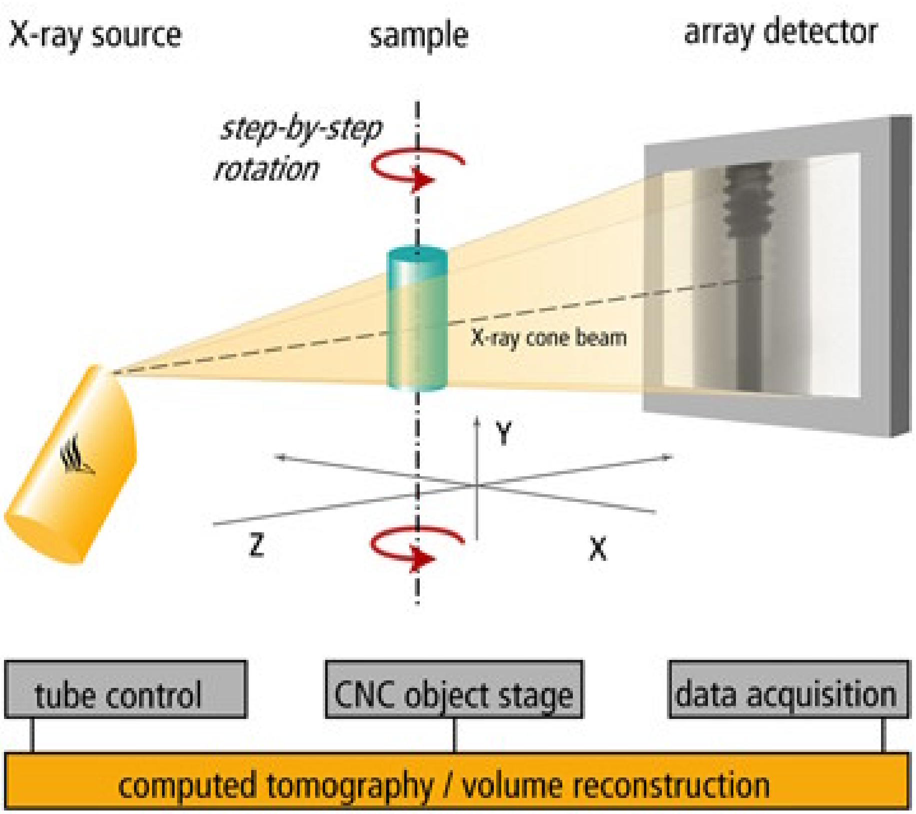
$$f_\Lambda = A^*\frac{\partial}{\partial s}g$$

Hilbert transform is non-local:

$$Hg(s) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{g(t)}{s-t}dt$$

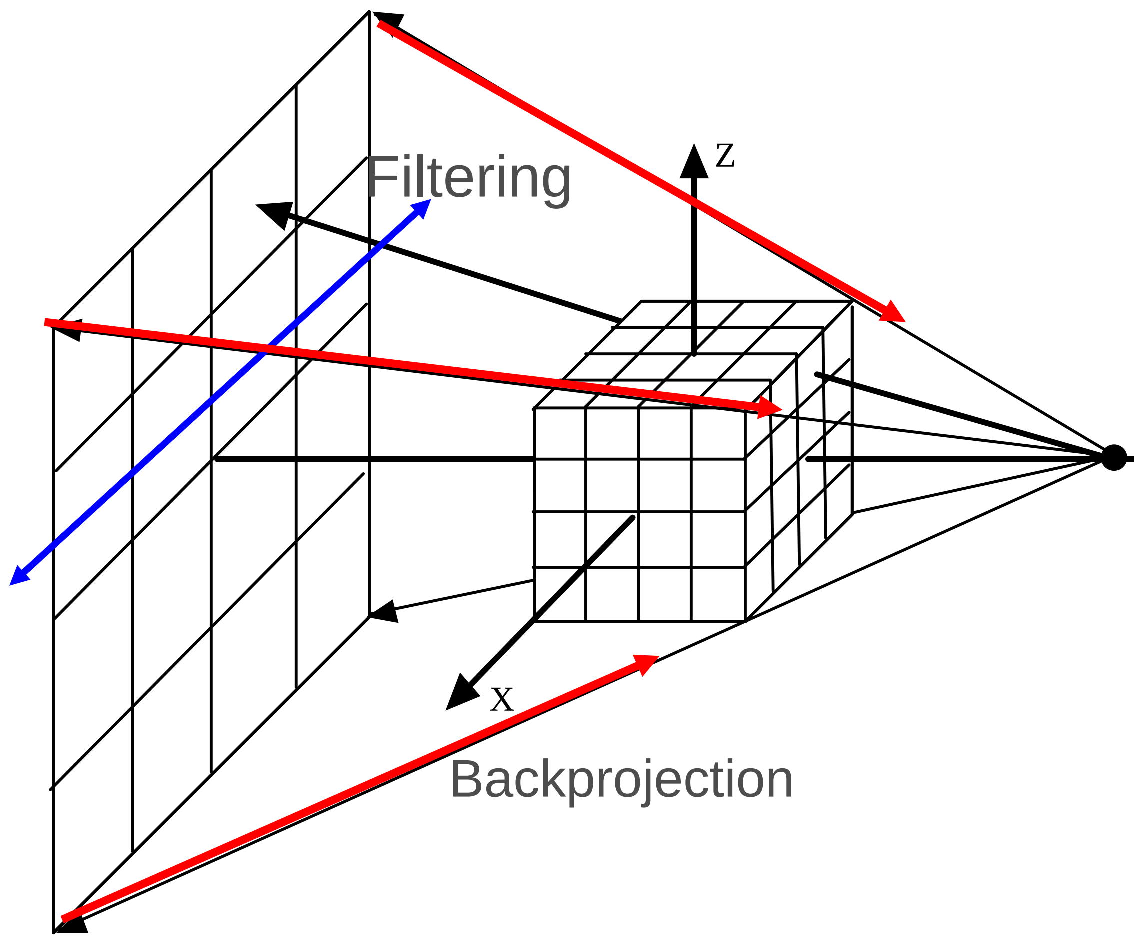
Cone-beam geometry (Microfocus x-ray tube)

Feldkamp algorithm with a circular orbit

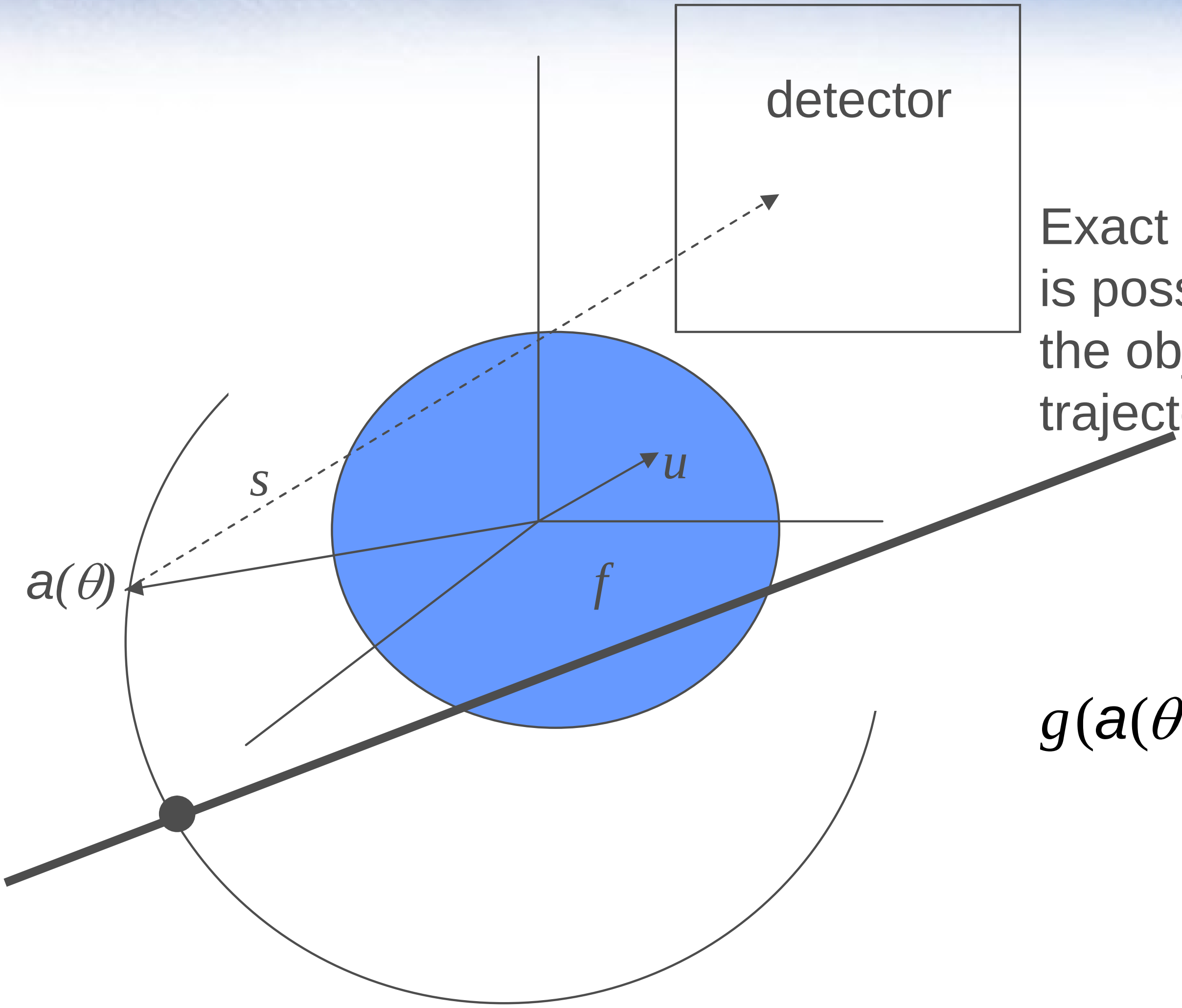


$$f = \int_0^{\pi} q * g_\theta \, d\theta$$

Feldkamp, Davis, Kress, 1984



Kirillov-Tuy condition

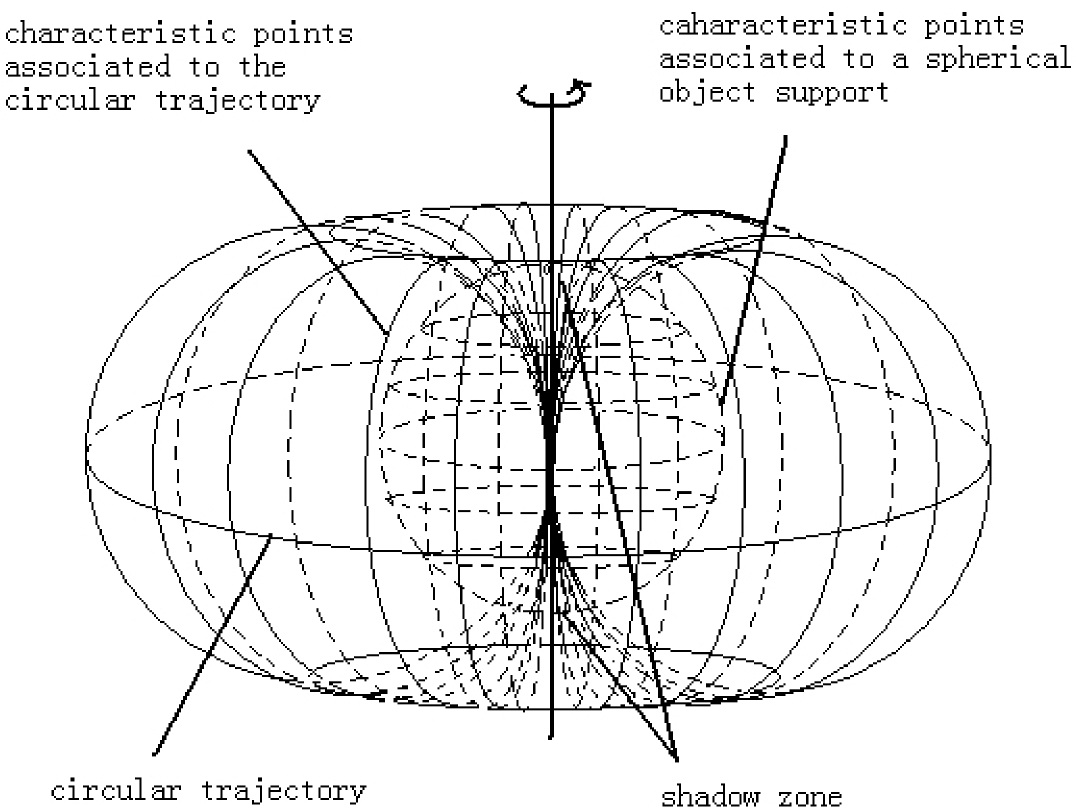


Exact 3D reconstruction is possible if every plane through the object intersects the source trajectory at least once

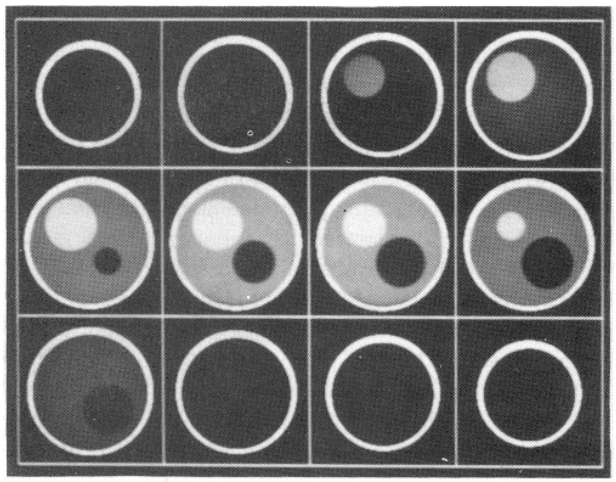
$$g(a(\theta),u) = \int_0^\infty f(a(\theta)+su)\,ds$$

x-ray source trajectory; parametrized as *a(θ)*

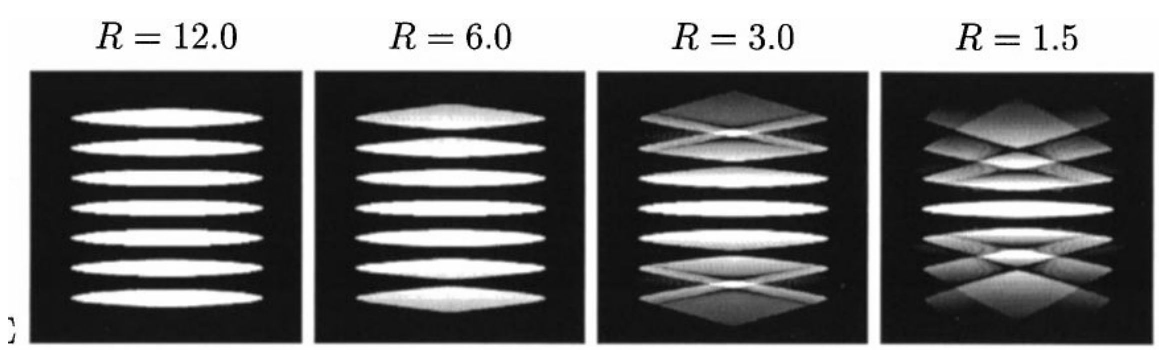
Circular source orbit: artifacts



Slices of 3D reconstruction
of a phantom (cone angle 30 deg):

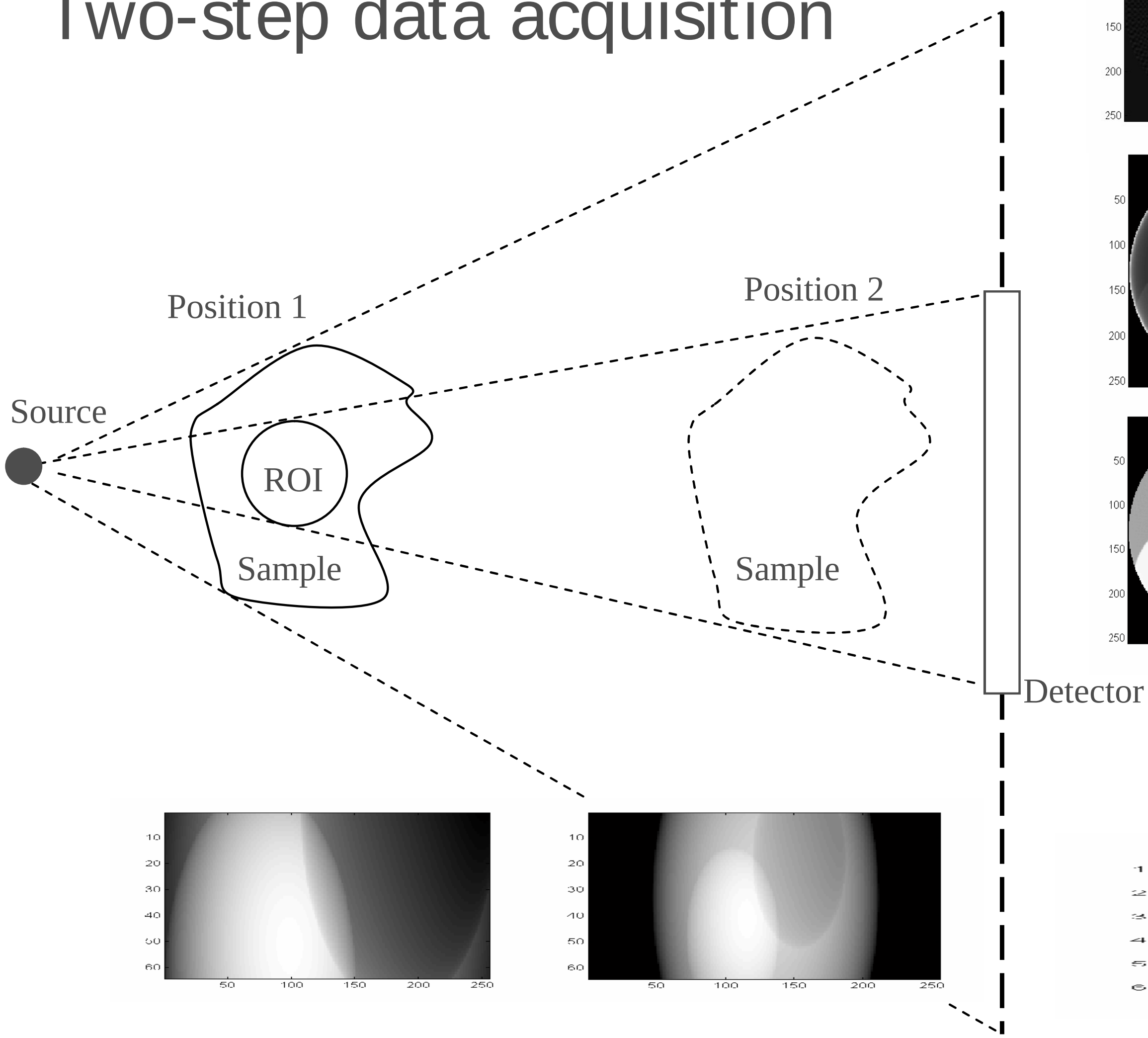


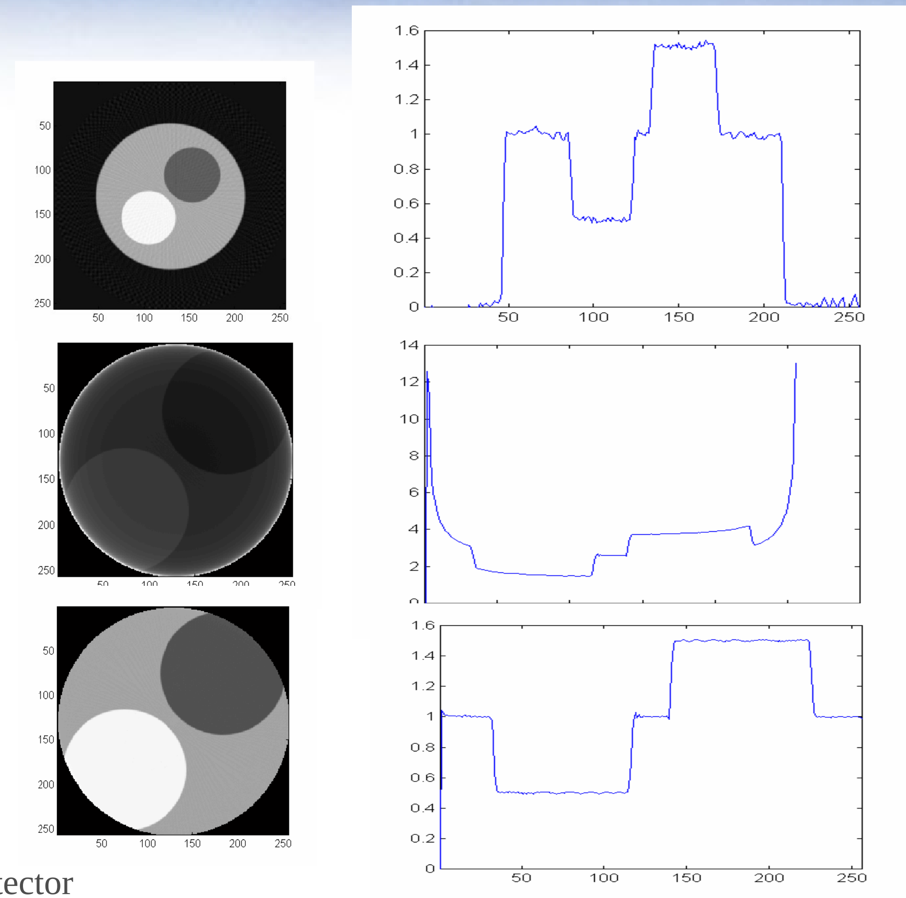
Bronnikov 1995, 2000

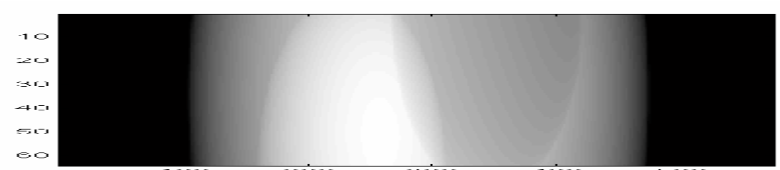


ROI reconstruction

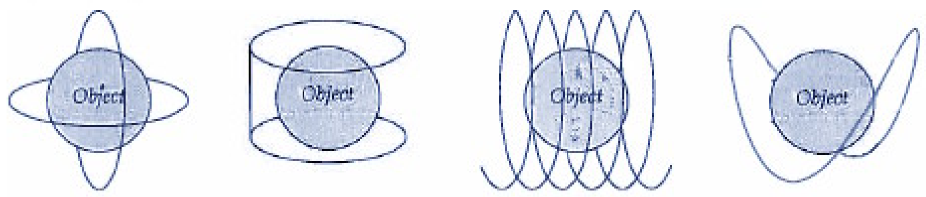
Two-step data acquisition



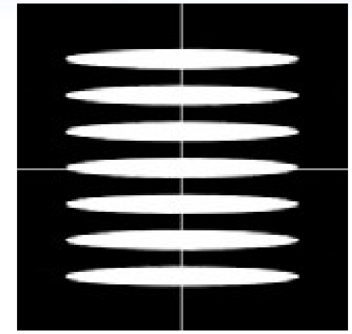


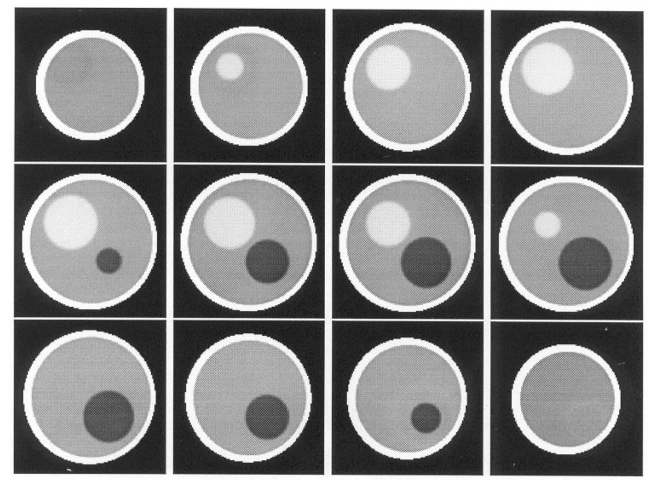


Non-planar source orbits



Non-planar
3D reconstructions
of a phantom:



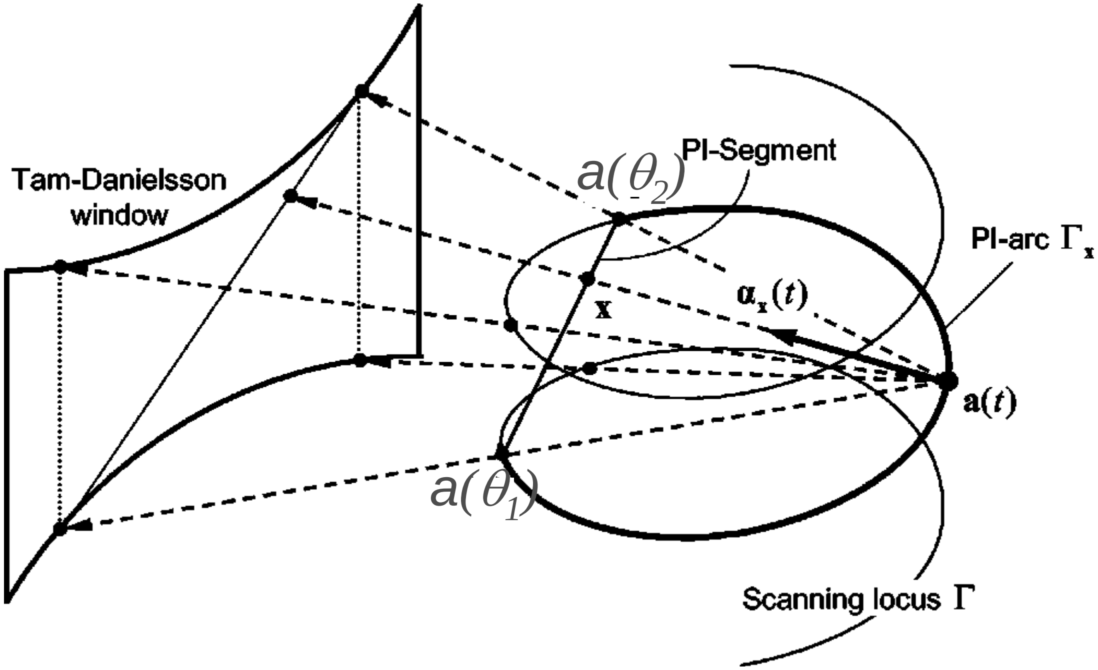


- two orthogonal circles
- two circles and line
- helix (most feasible mechanically)
- saddle

Non-planar orbits satisfy the Kirillov-Tuy condition,
but special reconstruction algorithms are required

Katsevich algorithm for a non-planar source orbit

Katsevich, 2002



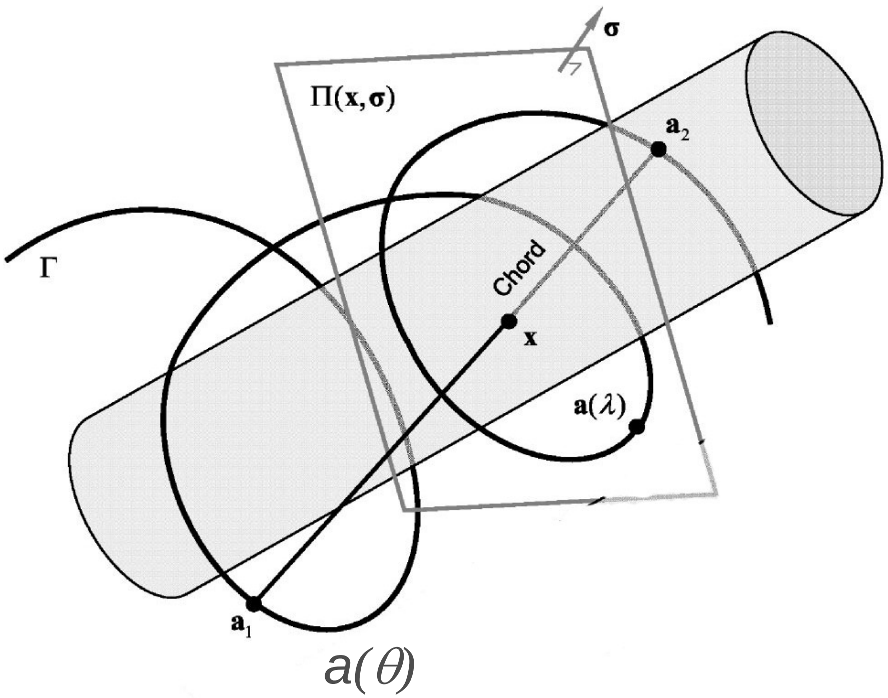
$$f = \frac{1}{2\pi} A^* H_\pi \frac{\partial}{\partial\theta} g$$

1. Differentiation of data
2. Hilbert transform along the filtration lines inside the Tam-Danielson window
3. Backprojection

PI-line (“segment”, “chord”) between $a(\theta_1)$ and $a(\theta_2)$: $\theta_2 - \theta_1 = 2\pi$

Helix: $$\mathbf{a}(\theta) = \left(R\cos\theta,\, R\sin\theta,\, \frac{h\theta}{2\pi} \right)^T$$

BPF algorithms for ROI reconstruction



$$f = \frac{1}{2\pi} H_\pi A_\pi^* \frac{\partial}{\partial\theta} g$$

1. Differentiation of data
2. Backprojection onto the PI chord (locality!)
3. Hilbert transform along the PI chord

Using that f has the finite support and

$$HHg = -g$$

Zou, Pan, Sidky, 2005 derived:

$$f = \frac{1}{2\pi} H_{\pi,\mathrm{a1-a2}}^{-1} A_\pi^* \frac{\partial}{\partial\theta} g$$

Ultra-fast implementation

- Graphic card (GPU)
- CPU

Reconstruction of a 512x512x512 image from 360 projections:

CPU (~2 GHz) :	Single	Dual core	Quad core	Twin quad-core
Time :	~80 sec	~40 sec	~20 sec	~10 sec

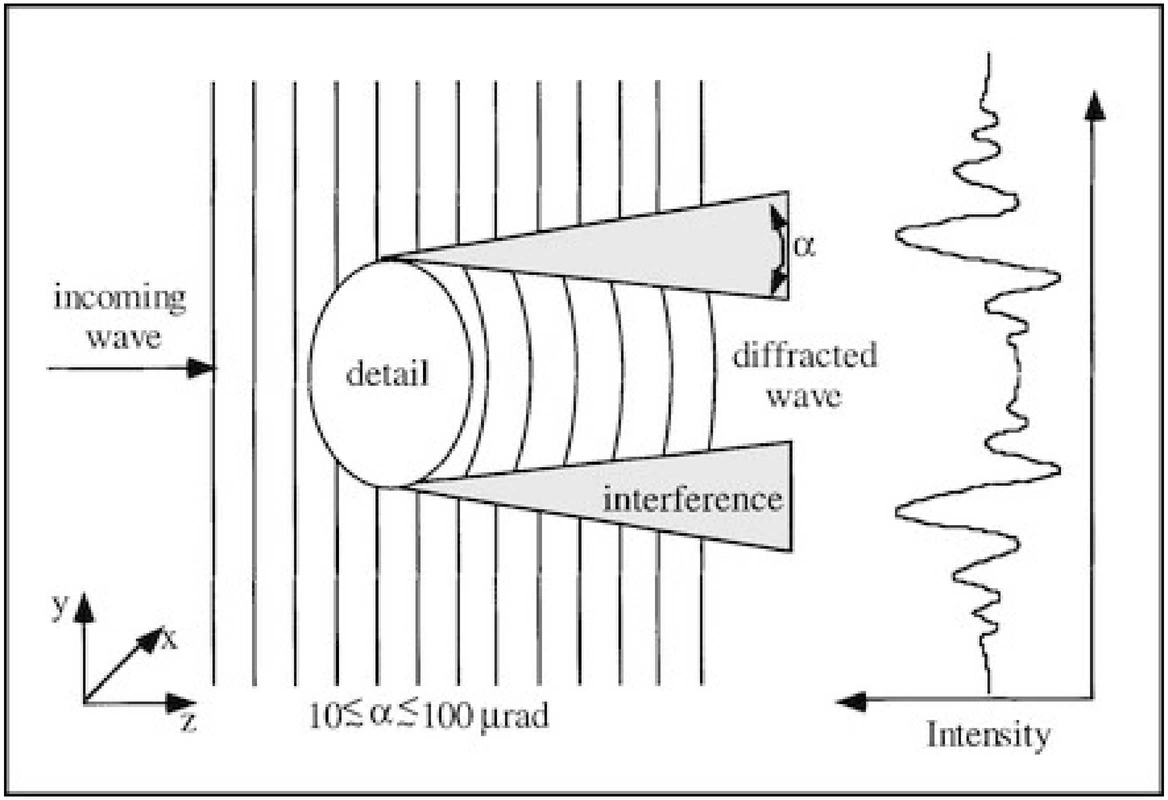
Reconstruction of a 1024x1024x1024 image from 800 projections:

CPU (~2 GHz) :	Dual core	Twin quad-core
Time :	~480 sec	~120sec

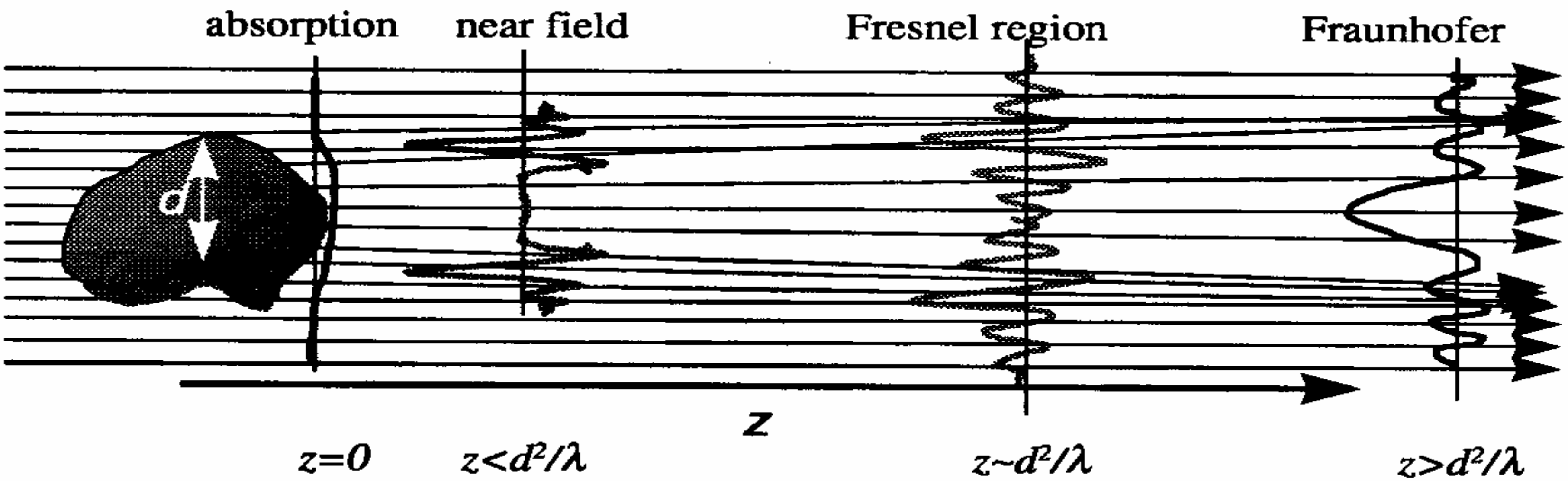
Phase-contrast microtomography (Free propagation mode)

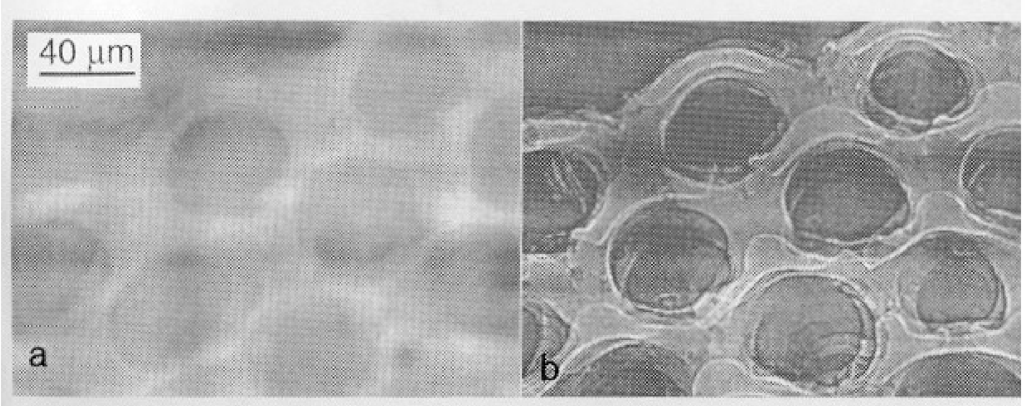
Phase contrast

Interference of the phase-shifted wave with the unrefracted waves



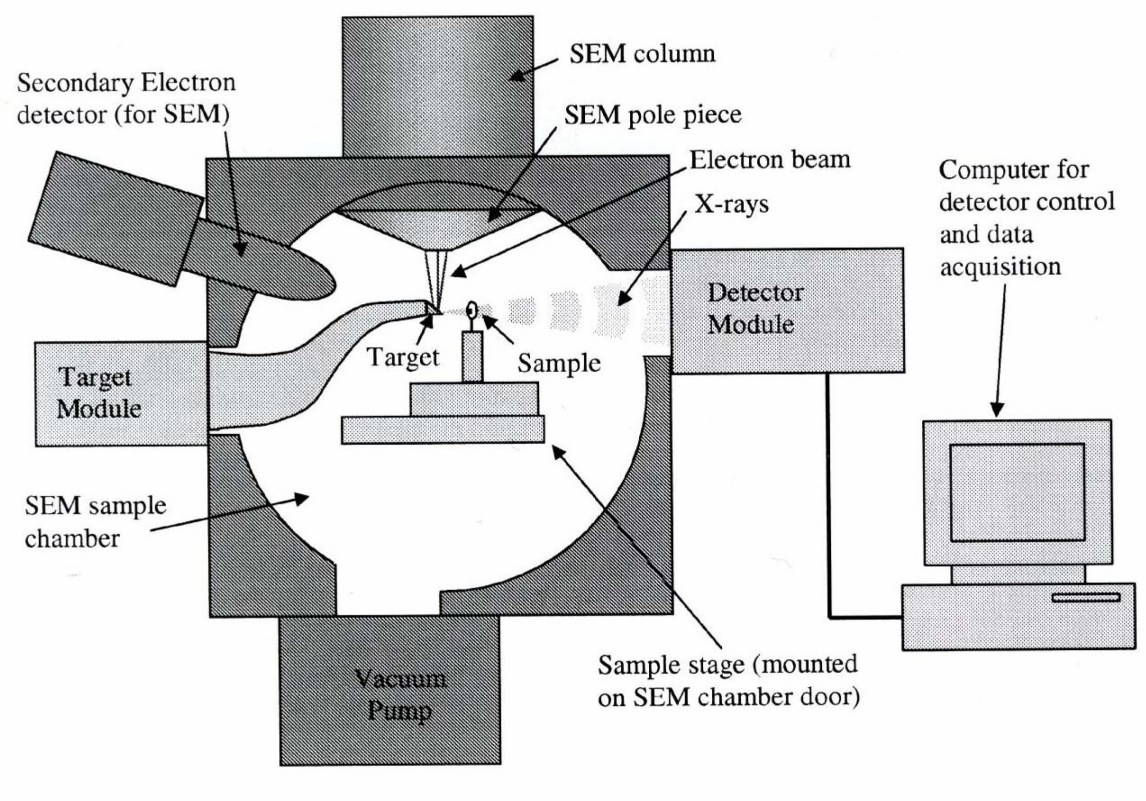
Inline phase-contrast imaging





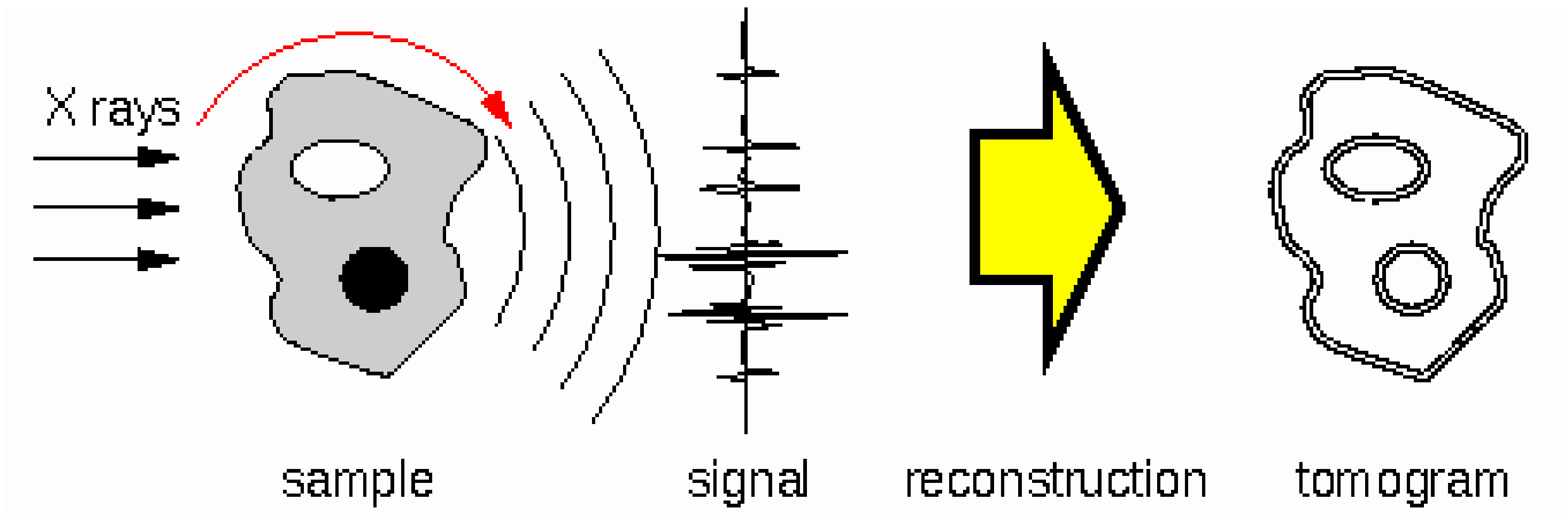
Snigirev *et al*, 1995

Polychromatic x-ray phase contrast



Wilkins *et al*, 1996

Phase-contrast tomography with Radon inversion: edges



Inverse problem of phase-contrast microtomography

Object function: $f = n - 1$

$$\text{find} \quad f(x_1, x_2, x_3) \quad \text{from} \quad I_\theta^z(x, y), \quad 0 \le \theta < \pi$$

- CTF (Cloetens *et al*, 1999)
- TIE (Paganin and Nugent, 1998)

} Phase retrieval, more than one detection plane

- Weak-absorption TIE (Bronnikov, 1999) FBP, single detection plane

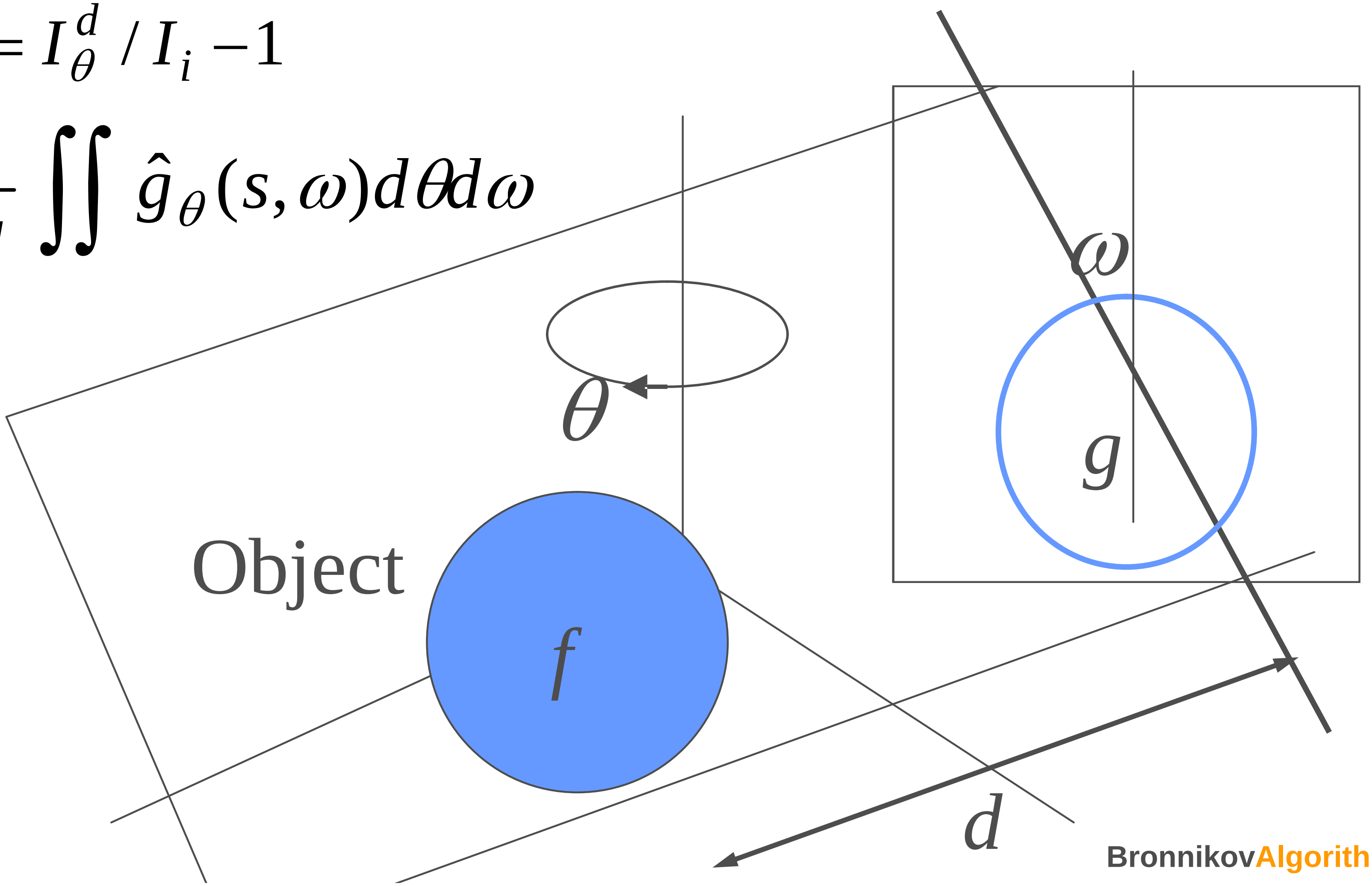
Radon transform solution of TIE

$$I_\theta^d(x,y) = I_\theta^0\left[1 - \frac{\lambda d}{2\pi}\nabla^2\varphi_\theta(x,y)\right]$$

Bronnikov, 1999

$$g_\theta(x,y) = I_\theta^d / I_i - 1$$

$$f = \frac{1}{4\pi^2 d}\iint \hat{g}_\theta(s,\omega)\,d\theta\,d\omega$$



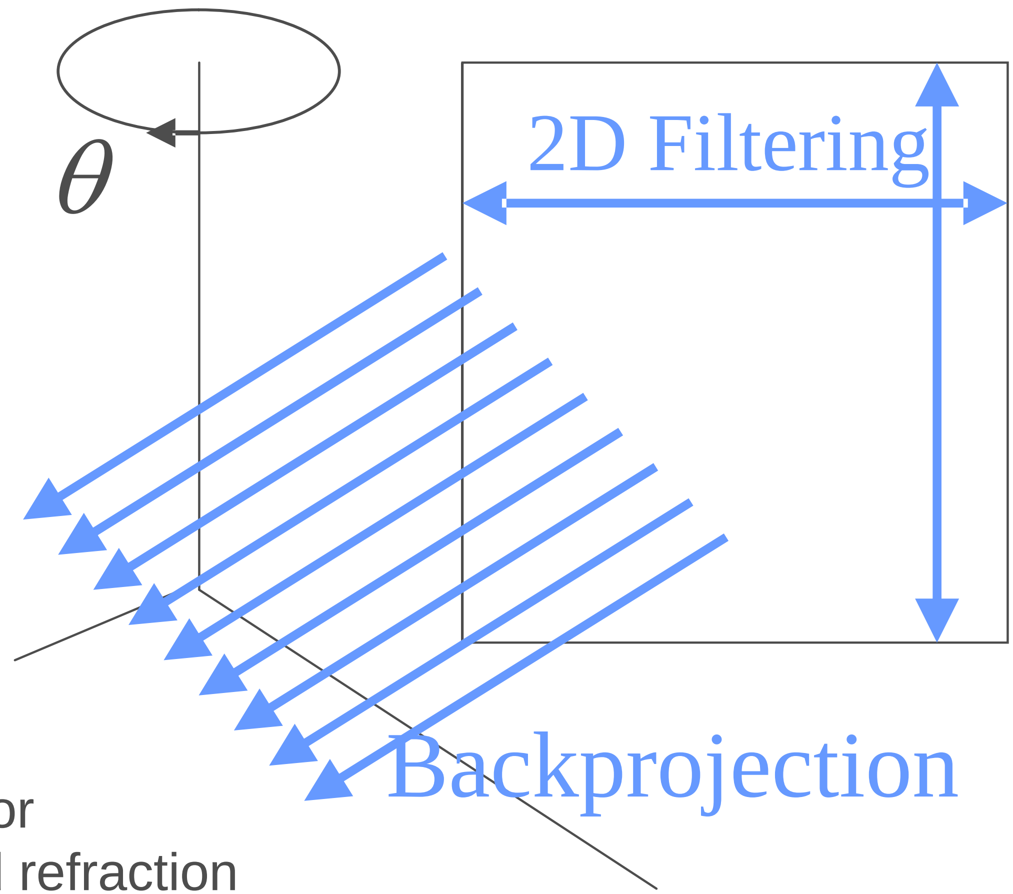
Phase-contrast reconstruction in the form of the FBP algorithm

Bronnikov, 1999, 2002, 2006

$$f = \frac{1}{4\pi^2 d} \int_0^{\pi} q**g_\theta d\theta \qquad q = \frac{|y|}{x^2 + y^2}$$

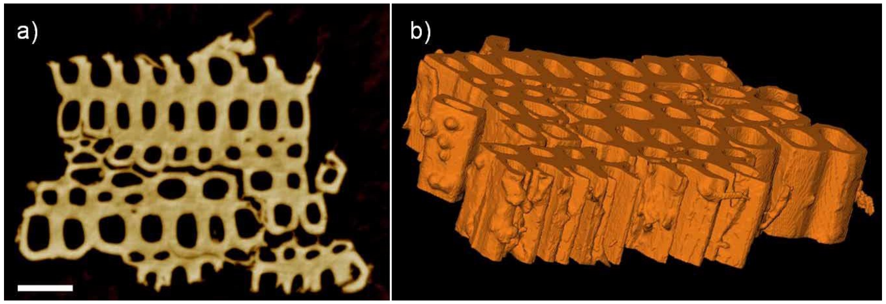
$$Q = \frac{|\xi|}{\xi^2 + \eta^2}$$

$$Q_\alpha = \frac{|\xi|}{\xi^2 + \eta^2 + \alpha}$$



Gureyev *et al*, 2004: choice of α for linearly dependent absorption and refraction

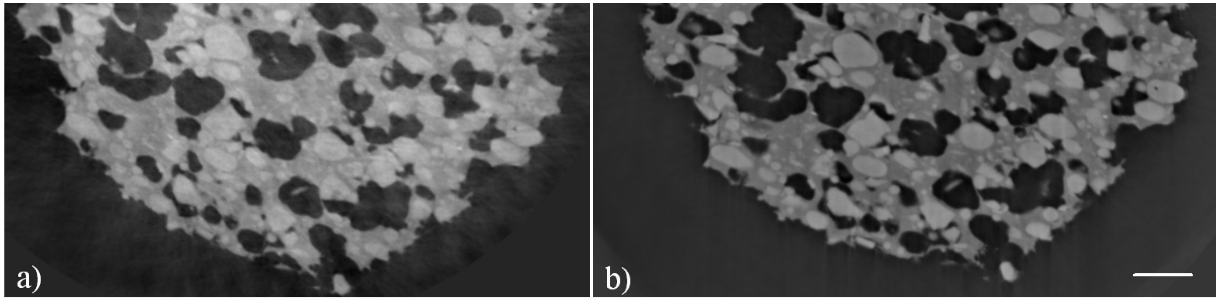
Implementation at SLS



Phase tomography reconstruction (a) and the 3D rendering (b) of a 350 microns thin wood sample using modified filter given in the Eq. (8). The length of the scale bar is 50 µm.

"MBA: Modified Bronnikov Algorithm"
Groso, Abela, Stampanoni, 2006

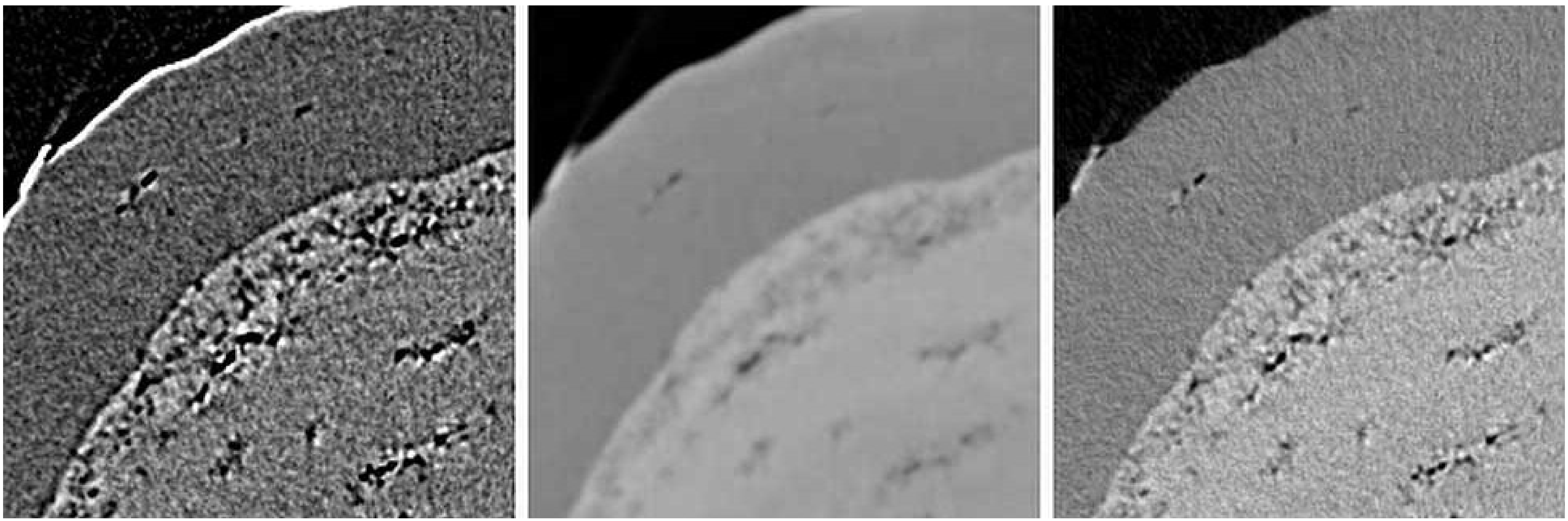
$$Q_\alpha = \frac{|\xi|}{\xi^2 + \eta^2 + \alpha}$$



Validation of the MBA method: (a) Phase tomographic reconstruction of sample consisting of polyacrylate, starch and cross-linked rubber matrix obtained using DPC and (b) using MBA. The length of the scale bar is 100 µm.

Implementation at Ghent University

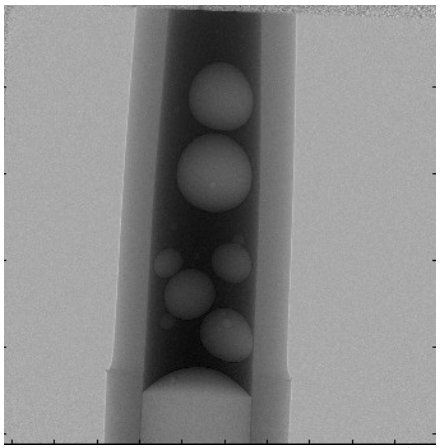
De Witte, Boone, Vlassenbroeck, Dierick, and Van Hoorebeke, 2009



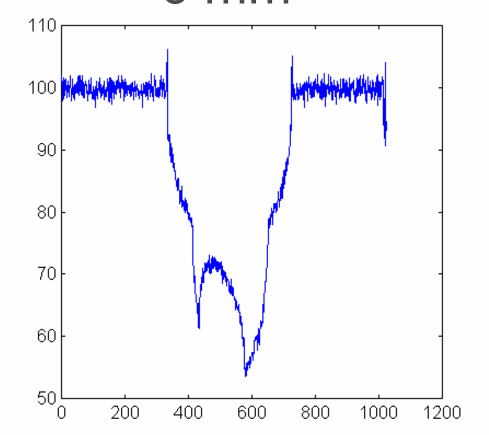
Radon inversion "Modified Bronnikov Algorithm" "Bronnikov-Aided Correction"

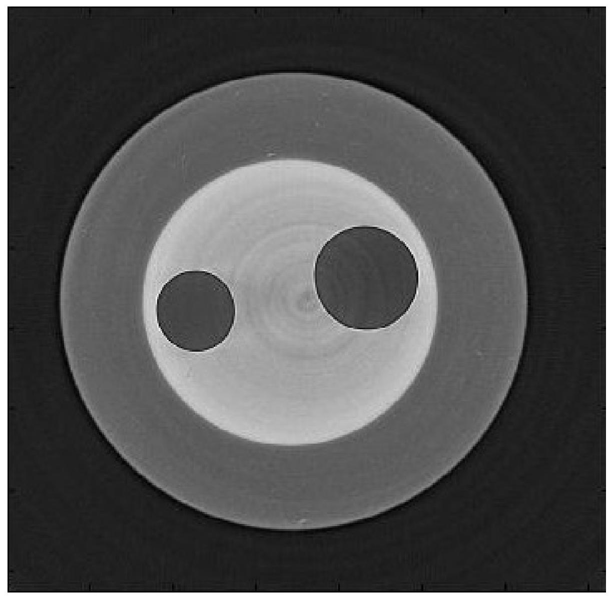
Polychromatic source, mixed phase and amplitude object

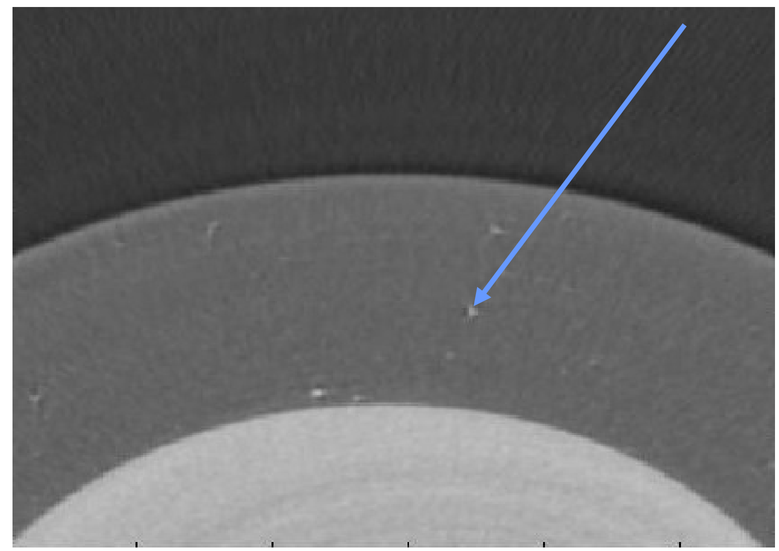
Air bubbles in epoxy,
relatively strong absorption:



6 mm







Reconstruction by the “Bronnikov Filter” with correction

Data provided by Xradia

Summary

- New developments in theory:
  - parallel-beam CT (synchrotron): the use of NFFT
  - cone-beam CT (microfocus): exact reconstruction with non-planar orbits; exact ROI reconstruction

    (Katsevich formula, PI line, Hilbert transform on chords)

- New developments in implementation:
  - ultra-fast 3D reconstruction on multicore processors
  - (1024^3 voxels within one-two minutes on a PC)

- New developments in coherent methods:
  - robust algorithms for 3D phase reconstruction
  - correction for the phase component